May 19, 2018 at 02:30

1. Intro. This little program outputs clauses that are satisfiable if and only if the graph g can be c-colored, given g and c. It differs from SAT-COLOR-LOG2 because it uses a shorter way to compare binary labels.

Suppose the graph has m edges and n vertices, and let $t = \lceil \lg c \rceil$. Then there are nt variables $v.k$, meaning that vertex v gets color $(v.1\ v.2\ \ldots\ v.t)_2$. The final bit $v.t$ is sometimes irrelevant; for example, when $c = 3$, colors 10 and 11 are considered to be the same, we can consider the three possible colors to be 00, 01, and $1*$. When $c = 5$ the five possible colors are 000, 001, $01*$, $10*$, and $11*$.

There are cm clauses of size $2t$ or $2t - 2$ to ensure that adjacent vertices don't share a color.

```
#include <stdio.h>
#include <stdlib.h>
#include "gb_graph.h"
#include "gb_save.h"
  int c;
  main(int argc, char *argv[])
  {
    register int i, k, kk, t;
    register Arc *a;
    register Graph *g;
    register Vertex *u, *v;
    ⟨ Process the command line 2 ⟩;
    for (t = 0; c > (1 ≪ t); t++) ;
    for (v = g→vertices; v < g→vertices + g→n; v++)
      for (a = v→arcs; a; a = a→next) {
        u = a→tip;
        if (u < v) ⟨ Generate clauses to keep u and v from having the same color 3 ⟩;
      }
  }
```

2. ⟨ Process the command line 2 ⟩ ≡

```
  if (argc ≠ 3 ∨ sscanf(argv[2], "%d", &c) ≠ 1) {
    fprintf(stderr, "Usage: %s foo.gb c\n", argv[0]);
    exit(-1);
  }
  g = restore_graph(argv[1]);
  if (¬g) {
    fprintf(stderr, "I couldn't reconstruct graph %s!\n", argv[1]);
    exit(-2);
  }
  if (c ≤ 0) {
    fprintf(stderr, "c must be positive!\n");
    exit(-3);
  }
  printf("~ sat-color-log3 %s %d\n", argv[1], c);
```

This code is used in section 1.

3. ⟨ Generate clauses to keep u and v from having the same color 3 ⟩ ≡

```
{
  for (k = c; k < c + c; k++) {
    for (i = t, kk = k; i; i--)
      if (i < t ∨ k ≥ (1 ≪ t)) {
        printf("␣%s%s.%d␣%s%s.%d", kk & 1 ? "~" : "", u→name, i, kk & 1 ? "~" : "", v→name, i);
        kk ≫= 1;
      }
    printf("\n");
  }
}
```

This code is used in section 1.

4. Index.

⟨ Generate clauses to keep u and v from having the same color 3 ⟩ Used in section 1.
⟨ Process the command line 2 ⟩ Used in section 1.

SAT-COLOR-LOG3