

May 19, 2018 at 02:30

1. Intro. This little program outputs clauses that are satisfiable if and only if the graph g can be c -colored, given g and c .

(It generalizes SAT-PIGEONS, which is the case where $g = K_m$ and $c = n$, and uses the “order encoding.”)

Suppose the graph has m edges and n vertices. Then there are $n(c - 1)$ variables $v < k$, meaning that vertex v gets color $< k$. And there are $n(c - 2)$ clauses of size 2 (to enforce the linear ordering of colors), plus mc clauses of size 4 (to ensure that adjacent vertices don’t share a color).

```
#include <stdio.h>
#include <stdlib.h>
#include "gb_graph.h"
#include "gb_save.h"
int c;
main(int argc, char *argv[])
{
    register int i, j, k;
    register Arc *a;
    register Graph *g;
    register Vertex *v;

    <Process the command line 2>;
    <Generate the ordering clauses 3>;
    <Generate the graph-coloring clauses 4>;
}
```

```
2. <Process line 2> ≡
if (argc ≠ 3 ∨ sscanf(argv[2], "%d", &c) ≠ 1) {
    fprintf(stderr, "Usage: %s foo.gb c\n", argv[0]);
    exit(-1);
}
g = restore_graph(argv[1]);
if (¬g) {
    fprintf(stderr, "I couldn't reconstruct graph %s!\n", argv[1]);
    exit(-2);
}
if (c ≤ 0) {
    fprintf(stderr, "c must be positive!\n");
    exit(-3);
}
printf("~sat-color-order %s %d\n", argv[1], c);
```

This code is used in section 1.

```
3. <Generate the ordering clauses 3> ≡
for (v = g-vertices; v < g-vertices + g-n; v++)
    for (k = 2; k < c; k++) printf("~%s<%d %s<%d\n", v-name, k - 1, v-name, k);
```

This code is used in section 1.

```

4. ⟨Generate the graph-coloring clauses 4⟩ ≡
  for (v = g-vertices; v < g-vertices + g-n; v++)
    for (a = v-arcs; a; a = a-next)
      if (a-tip > v)
        for (k = 1; k ≤ c; k++) {
          if (k ≡ 1) printf("%s< %d_~%s<%d\n", v-name, k, a-tip-name, k);
          else if (k < c) printf("%s< %d_~%s<%d_%s<%d_~%s<%d\n", v-name, k - 1, v-name, k,
            a-tip-name, k - 1, a-tip-name, k);
          else printf("%s<%d_%s<%d\n", v-name, k - 1, a-tip-name, k - 1);
        }

```

This code is used in section 1.

5. Index.*a*: 1.**Arc**: 1.*arcs*: 4.*argc*: 1, 2.*argv*: 1, 2.*c*: 1.*exit*: 2.*fprintf*: 2.*g*: 1.**Graph**: 1.*i*: 1.*j*: 1.*k*: 1.*main*: 1.*name*: 3, 4.*next*: 4.*printf*: 2, 3, 4.*restore_graph*: 2.*sscanf*: 2.*stderr*: 2.*tip*: 4.*v*: 1.**Vertex**: 1.*vertices*: 3, 4.

- ⟨Generate the graph-coloring clauses 4⟩ Used in section 1.
- ⟨Generate the ordering clauses 3⟩ Used in section 1.
- ⟨Process the command line 2⟩ Used in section 1.

SAT-COLOR-ORDER

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